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14. ABSTRACT: In this 33-month period this PI and his group developed new research directions and made progress in their fundamental and application research done previously for AFOSR.

The most significant project was to predict new effects in laser-excited atomic nano-structures and develop their theory(see corrugation growth[2,5,6,8] below). In a pilot PRL paper [2] This PI and his post-doc d. Volkov reported the discovery of fundamentally new phenomenon in the electrodynamics of interaction of light with atomic lattices. They revised the Lorentz-Lorenz theory of local fields, and showed the existence of new excitation, the so called locsitons. They opens up a possibility of new, faster and smaller, computer dielectric elements on a nano-scale, as well as extremely sensitive nano-scale bio-detectors. A detailed theory of one-dimensional [5] and two-dimensional cases [8] and nonlinear multi-stability [5,6,8] have been developed.

New results on evanescent field at nano-corrugated dielectric surface [3] continues the line of this PI's collaboration with the research group of the Univ. of Kioto, Japan. In [3] we predicted that, counterintuitively, the field at the bottom of laser-induced nano-grooves exceeds the intensity of incident laser light, which provides positive feedback for the experimentally observed corrugation growth

In a paper [4], this PI and his post-doc A. Pokrovsky predicted multi-MeV electron acceleration and zeptosecond electron bunching in extremely tightly focused laser beam, which is a continuation of their previous research on relativistic ponderomotive force.

Jointly with German and Italian colleagues, this PI developed the theory of multi-mode quantum interference in low-relativistic excitation of electrons, [1]. This 25-page long paper goes into intricate interactions in fundamental quantum mechanics, important in particular for potential applications to new computer technologies and cold atoms.

This PI demonstrated that nonlinear excitation in relativistic plasmas can result in giant "rogue" waves [9] supported by external pumping with only a small fraction of their power.

In a paper [7] on a single-particle motional oscillator powered by laser, this PI demonstrated that a heavy atom or a cluster trapped in an optical, opto-magnetic or Penning trap, can develop vibrational oscillations powered by the radiation forces induced by an off-resonance blue-detuned cw laser. The oscillations axis conserves its direction in space regardless of trap rotation, developing thus a "Foucault pendulum" even in non-gravitational environment; it can be used for inertial navigation in space. Within less than a year after that, an experimental observation of this effect was reported in "Nature Physics" by a group of Prof. T. W. Hanch, Nobel prize winner.

Archival (journal) publications during reported period: [1] I. Marzoli, A. E. Kaplan, F. Saif, and W. P. Schleich, "Quantum carpets of slightly-relativistic particle", Fortschritte der Physik, v. 56, No. 10, pp. 967-992 (2008) ; [2] A. E. Kaplan and S. N. Volkov, "Nanoscale stratification of local optical fields in low-dimensional atomic lattices", Phys. Rev. Lett., v. 101, 133902(1-4) (26 Sept. 2008) ; [3] S. N. Volkov, A. E. Kaplan, and K. Miyazaki, "Evanescent field at nanocorrugated dielectric surface", Appl. Phys. Letters v. 64, 041104(1-3) (26 Jan. 2009); [4] A. E. Kaplan and A. L. Pokrovsky, "Laser Gate: Multi-MeV electron acceleration and zeptosecond e-bunching", Optics Express, v. 17, pp. 6194-6202 (13 Apr. 2009) ; [5] A. E. Kaplan and S. N. Volkov, "Local fields in nano-lattices of strongly-interacting atoms: nano-strata, giant resonances, magic numbers, and optical bistability", in "Progress in Physical Sciences" ("Uspekhi Fizicheskikh Nauk", Russia), UFN, v. 179, pp. 539-547 (May 2009); in "Physics -- Uspekhi" (English translation of above), v.52, pp. 506-514 (May 2009).; [6] A. E. Kaplan and S. N. Volkov, "Nano-stratification of optical excitation in self-interacting one-dimensional arrays" Phys. Rev. A, v. 79, 053834(1-16) (May 18, 2009) ; [7] A. E. Kaplan, "Single-particle motional oscillator powered by laser" Optics Express, v. 17, 10035-10043 (8 May, 2009) ; [8] S. N. Volkov and A. E. Kaplan, "Local-field excitations in two-dimensional lattices of resonant atoms", Phys. Rev. A, v. 81, 043801(1-10) (April 1, 2010); [9] A. E. Kaplan, "Optical Multi-hysteresises and "Rogue Waves" in Nonlinear Plasma" pending; (arXiv:1003.2837v1, PDF file, March 15'10).

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Optical Multi-hysteresises and "Rogue Waves" in Nonlinear Plasma

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(Dated: December 30, 2009)

An overdense plasma layer irradiated by an intense light can exhibit dramatic nonlinear-optical effects due to a relativistic mass-effect of free electrons: highly-multiple hysteresises of reflection and transition, and emergence of gigantic "rogue waves". Those are trapped quasi-soliton field spikes inside the layer, sustained by an incident radiation with a tiny fraction of their peak intensity once they have been excited by orders of magnitude larger pumping. The phenomenon persists even in the layers with "soft" boundaries, as well as in a semi-infinite plasma with low absorption.

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Diverse wave-related phenomena exhibit a common critical behavior: a dramatic transition at a crossover point from a traveling wave in the underlying (semi-infinite) medium to a non-propagating, evanescent wave that carry no energy. The crossover occurs in optics at the angle of total internal reflection at a dielectric interface, at a laser frequency near either a plasma frequency, or a waveguide cut-off frequency, or band-gap edge of a material, including BEC; in quantum mechanics for electron scattering at the energy close to a potential plateau, etc. It can be of great significance to *nonlinear* optics: a nonlinear refractive index can cause a phase-transition-like effect, since a small light-induced change may translate into a switch from full reflection to full transmission, resulting in a huge hysteresis. Predicted in [1] for nonlinear interfaces, it was explored experimentally in [2], with an inconclusive outcome, with some of the experiments (including the latest [2c]) showing a clear hysteresis, while [2b] showing none (see also below). 2D numerical simulations were not well suited then for modeling hysteresis; their very formulation excluded multivalued outcome by using single-valued boundary conditions.

In this Letter we discover, however, that even the most basic, 1D-case, reveals a large and apparently little known phenomenon of highly-multistable nonlinear EM-propagation and the emergence of trapped "rogue" (R) waves, with intensity exceeding the incident one by orders of magnitude. They may be released by plasma expansion, in a phase-like transition, as in a boiling liquid. Even a slight nonlinearity due to the most fundamental mechanism – relativistic (RL) mass-effect of free electrons – suffices to initiate the effect. Multiple (up to hundreds) hysteretic jumps between almost full reflection and full transparency may occur as the laser intensity is swept up and down. We treat the problem here in the context of ideal plasma on account of recent interest in high-intensity laser-plasma interactions and fundamental nature of RL-nonlinearity, but all approaches are valid for other crossover problems. Temporal RL-solitons have been considered in detail in the literature [3]; close to those of underdense plasma are the so called Bragg or band-gap solitions [4], including the ones in BEC [5]. The difference in this work is made by multi-hysteresises (and *standing, unmoving* quasi-solitons instead of propagating ones) due to self-induced retro-reflection. The remarkable new property is that for the *same incident power* an EM-wave can penetrate into a nonlinear material to *different depths* – varying by orders of magnitude – depending of the history of pumping. We assume a stationary, *cw*, or long pulse mode, and use only RL-nonlinearity in a cold plasma. While this model is greatly simplified *vs* various kinetic approaches, it allows us to keep the basic features necessary to elucidate new results, and have the theory applicable to other systems.

Even a few-λ-thick plasma layer can produce the effect, so that the absorption of light (including nonlinear self-focusing) would not affect the propagation significantly. Furthermore, while abrupt boundaries contribute to the effect, they are not essential: a layer with "soft" boundaries still exhibits all the major features of the phenomenon. The phenomenon persists even for *semi-infinite* plasma with a small absorption, which also develops a strong retro-reflection. The "Sommerfeld condition" (no wave comes from the "infinity") is to be revisited here: a wave *is back-reflected* deep inside the plasma and returns to the boundary. Imposed on a forward wave, it results in a semi-standing wave, trapped quasi-soliton R-waves, and multistability, same as in a finite layer. The energy accumulated in R-waves and excited free electrons, can then be released if plasma density reduces, as it may happen, e. g. in an astrophysical environment.

The wave propagation here is governed by the same so called nonlinear Klein-Gordon-Fock equation:

$$[\nabla^2 - \partial^2/(\mathbf{v}\partial t)^2]\psi = k_0^2 f(|\psi|^2)\psi; \quad f(0) = 1. \qquad (1)$$

where $f(|\psi|^2)$ is a function responsible for nonlinearity. Here, a generic variable, ψ, could be a scalar (e.g. a wave function in RQM), or a field vector in EM-wave propagation, $\mathbf{v}$ is a scale velocity, ($\mathbf{v} = \mathbf{c}$ for plasma and in RQM), and k_0^{-1} is a spatial scale of the problem, (for RQM, $k_0 = m_0 c/\hbar = 2\pi/\lambda_C$, where λ_C is the Compton wavelength, and m_0 is the rest-mass of an electron); for plasma $k_0 = \omega_{pl}/c$, where ω_{pl} is a plasma frequency due to free-charge density, ρ_e, and in the X-ray physics, $\omega_{ph} =$

$E_{ph}/\hbar$ is related to photo-ionization limit, E_{ph}, of atoms. For an ω-monochromatic wave $\psi = \vec{\mathcal{E}}(\vec{r})\ e^{-i\omega t}/2 + c.c.$, where $\vec{\mathcal{E}}(\vec{r})$ is a complex amplitude, and using $u^2 = |\mathcal{E}|^2$, (1) is reduced to a nonlinear Helmholtz equation:

$$\nabla^2 \vec{\mathcal{E}} + k^2 \epsilon(u^2)\vec{\mathcal{E}} = 0; \quad \epsilon_{pl} = 1 - \omega_{pl}^2(u^2)/\omega^2 \qquad (2)$$

where $k = \omega/c$, and $\omega_{pl}^2 = 4\pi e\rho_e/m$, m – electron mass, and $\mathcal{E} = E/E_{nl}$ with E_{nl} being some characteristic nonlinear scale; for a RL-mass-effect it is $E_{rl} = \omega m_0 c/e$.

In general, nonlinear ϵ may have various origins: a varying ionization rate, plasma waves, ponderomotive force, etc. Assuming fully ionized gas, $\rho_e = const$, and a circularly-polarized wave, $\mathcal{E}(\zeta)\ (\hat{e}_x + i\hat{e}_y)\ e^{-i\omega t}/2 + c.c.$, that has very negligible high-harmonics generation and minimal longitudinal plasma waves excitation, the most basic remaining source of nonlinearity is a field-induced RL mass-effect of electron: $m = m_0\gamma$, with a relativistic factor $\gamma = \sqrt{1 + (p/m_0 c)^2} = \sqrt{1 + u^2}$ [see e. g. [6]) where p is the momentum of electron, so that

$$\epsilon_{rl} = 1 - [\nu^2\gamma(u^2)]^{-1} \qquad \text{with} \quad \nu = \omega/(\omega_{pl})_0 \qquad (3)$$

where $(\omega_{pl})_0$ is a linear plasma frequency with $m = m_0$. Since $m = m(u^2)$, a single electron exhibits large hysteretic cyclotron resonance predicted in [6a] and observed in [7]. The mass-effect has also became one of the major players of light-plasma interaction [3], e. g. in RL self-focusing, and in acceleration of electrons by the beat-wave and wake-field. The EM-propagation could also be accompanied by RL-intrinsic bistability [8].

In a 1D-case, letting a plane EM-wave propagate in the z-axis, we have $\nabla^2 = d^2/dz^2$. For a boundary between two dielectrics, a EM-wave incident from a dielectric with $\epsilon_{in} > 0$ under the angle θ onto a material of $\epsilon_{NL} > 0$, ϵ in (2) is replaced by $\epsilon_{in}[\epsilon_{NL}(\omega)/\epsilon_{in}(\omega) - \sin^2\theta]$. For a mw waveguide with a critical frequency ω_{wg}, ϵ in (2) is replaced by $\epsilon_{wg}(1 - \omega_{wg}^2/\omega^2)$. The crossover point is attained at $\epsilon = 0$. In this approximation, (2) reduces to

$$\mathcal{E}'' + \epsilon(\zeta, u^2)\mathcal{E} = 0, \qquad (4)$$

where $\zeta = kz$, and "prime" denotes $d/d\zeta$; in general, we do not assume ϵ $uniform$ in ζ-axis. In a weakly-nonlinear media one can break the field into counter-propagating traveling waves and find their amplitudes *via* boundary conditions. However, near a crossover point one in general cannot distinguish between those waves. To make no assumptions about the wave composition, we represent the field using real variables u, and phase (eikonal), ϕ, as $\mathcal{E} = u(\zeta)\exp[i\phi(\zeta)]$. Since $\mathcal{E}$ is in general complex, while $\epsilon = \epsilon(u^2)$, Eq. (4) is isomorphous to a 3-rd order eqn for u; yet, it is fully integrable in quadratures. Its first integral is a scaled momentum flux of EM-field, $P \equiv u^2\phi' = inv$. In a lossless media P is conserved over the entire space $\zeta < \infty$, even if the medium is non-uniform, multi-layered, linear and/or nonlinear, etc. If a layer borders a dielectric of $\epsilon = \epsilon_{ex}$ at the exit, we have $P = u_{ex}^2\sqrt{\epsilon_{ex}}$, where u_{ex}^2 is the exit wave intensity. Eq. (2) is reduced then to a 2-nd order equation for u:

$$u'' + u[\epsilon(\zeta, u^2) - P^2/u^4] = 0, \qquad (5)$$

which makes an unusual yet greatly useful tool. By addressing only a real amplitude, while using flux P as a parameter, (5) is nonlinear even for a *linear propagation*, yet is still analytically solvable if a density ρ_e is uniform across the layer ($\partial\epsilon/\partial\zeta = 0$). A full-energy-like invariant of (5) is ${u'}^2/2 + U(u^2) = W = inv$, with $U = [\int_0^{u^2} \epsilon(u^2)\ d(u^2) + P^2/u^2]/2$, where ${u'}^2/2$ is "kinetic", and U – "potential" energies. For a RL-nonlinearity (3), $U(u^2) = (u^2 + P^2/u^2)/2 - [\gamma(u^2) - 1]/\nu^2$. Here W is a scaled free EM energy density of ϵ-nonlinear medium [9] $W = c[H^2 + \int \epsilon d(E^2)]/(2E_{rl}^2)$, where H is magnetic field. If a layer exit wall is a dielectric, one has $W = U(u_{ex}^2)$, since then $u'_{ex} = 0$ (see below). For a metallic mirror, $W = {u'_{ex}}^2/2$, since now $u_{ex} = 0$; and $W = 0$ for an evanescent wave in a semi-infinite medium. The implicit solution for spatial dynamics of u in general case is found now as $\zeta = \int \{2[W - U(u^2)]\}^{-1/2} du$.

Boundary conditions at the borders with linear dielectrics at the entrance, $\zeta = 0$, with ϵ_{in}, and at the exit, $\zeta = d$, with ϵ_{ex}, result in complex amplitudes of incident, $\mathcal{E}_{in}$, and reflected, $\mathcal{E}_{rfl}$, waves at $\zeta = 0$:

$$\mathcal{E}_{in,rfl} = [u \pm \epsilon_{in}^{-1/2}(P/u - iu')]/2; \qquad (6)$$

where "+" corresponds to $\mathcal{E}_{in}$, and "−" – to $\mathcal{E}_{rfl}$, and the exit point, $\zeta = d$: $u = \mathcal{E}_{ex} \equiv u_{ex}$; $u' = 0$; and $\phi' = \sqrt{\epsilon_{ex}}$. A condition $P = 0$ corresponds to full reflection, resulting in either strictly standing wave, or nonlinear evanescent wave in a semi-infinite plasma, in particular in a "standing" soliton-like solution (see below). If $\epsilon(u^2 = 0) < 0$, there are no *linear* traveling waves; yet a purely traveling *nonlinear* wave may exist at sufficiently strong intensity $u^2 = u_{trv}^2 = const$, such that $\epsilon(u_{trv}^2) > 0$:

$$\mathcal{E}_{trv} = u_{trv}\exp(i\phi'\zeta), \quad \phi' = \pm\sqrt{\epsilon(u_{trv}^2)}, \qquad (7)$$

propagating either forward (+) or backward (−). However, if $\epsilon(u = 0) < 0$, it is strongly unstable. A non-periodic solution of (5) with $P = 0$ is a nonlinear evanescent wave that forms a standing, trapped soliton. In low-RL case, one needs a small detuning from crossover point, $\delta \equiv \nu - 1 \ll 1$, to attain the effect at low laser intensity, $u^2 \ll 1$, so that the the dielectric constant (3) is Kerr-like and small: $\epsilon_{rl} \approx -2\delta + u^2/2$, $|\epsilon_{rl}| \ll 1$. A full solution of (5) with $P = 0$ and $u \to 0$ at $\zeta \to \infty$ yields then a standing soliton with a familiar intensity profile:

$$u^2 = 8\delta/\cosh^2[(\zeta - \zeta_{pk})\sqrt{2\delta}] \qquad (8)$$

where the peak location ζ_{pk} is an integration constant. For an arbitrary frequency, $\nu < 1$, the soliton peak intensity is $u_{sol}^2 = 4(1 - \nu^2)/\nu^4$, instead of 8δ as in (8);

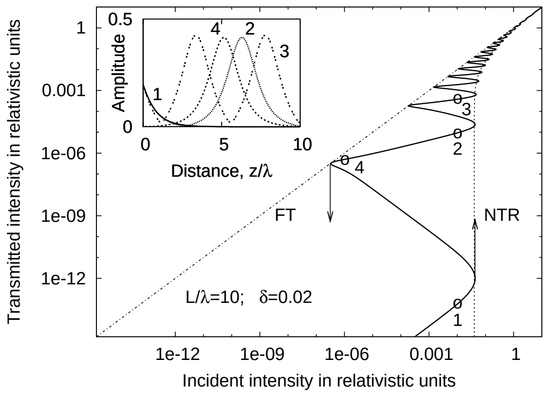

FIG. 1: Hysteretic transmission of light through a plasma layer of thickness L. **FT** and **NTR** are full transparency and near total reflection limits. Points 1, 2, 3 mark a linear evanescent wave, 1-st, and 2-nd upper stable states respectively, and 4 – a R-wave sustained by a very low pumping. Arrows indicate direction of jumps within the lowest hysteretic loop. Inset: spatial amplitude profiles of waves corresponding to points 1-4 in the main plot.

$\epsilon(u_{pk}^2) = (1-\nu^2)/(2-\nu^2) \geq 0$. When $\nu^2 < 1/2$, it is a strongly-RL soliton, $u_{sol}^2 \gg 1$, and its peak narrows down to a half-wave: $u^2 \approx u_{sol}^2 \cos^2(\zeta - \zeta_{pk})$ at $u^2 > 1$.

In a finite layer one has a mix of standing/evanescent and traveling waves, with $u_{min}^2 = u_{ex}^2 = P > 0$. A full integration of (5) with nonlinearity $\epsilon_{rl} \approx -2\delta + u^2/2$ yields then elliptic integrals of imaginary argument of the first kind; more importantly, eq. (5) and its invariant avails themselves to detailed analysis. The numerical simulations are needed, however, to find a solution for (a) strongly-RL field [using (5) and its integrals], or (b) non-uniform plasma density in (5), or(c) plasma with absorption [eq. (4)]. It then is found by an "inverse propagation" procedure, whereby we essentially back-track the propagation from a purely traveling exit wave back to the entrance. One sets first a certain magnitude of $u_{ex}^2 = P$, $u_{ex}' = 0$ at the exit, numerically computes an amplitude profile $u(\zeta)$ back to the entrance and incident and reflected intensities u_{ex}^2 and u_{rfl}^2 using (6), and then maps u_{ex}^2 and u_{rfl}^2 *vs* incident intensity, u_{in}^2. A data set $u_{in}^2(P)$ and $u_{rfl}^2(P)$ for any given P is found then with a single run, *vs* a so called multi-shooting commonly used in search of solution with conditions set at two boundaries. This provides a very fast numerical simulation *vs* multi-shooting; besides, the latter one is very unreliable when dealing with apriory unknown number of multi-solutions.

For a fully-RL simulation with a $L = 10\lambda$, where L is the layer thickness, Fig. 1 show the emergence of large number, N_{hs}, of huge hysteretic loops of the transmission (same as in reflection, not shown here), which bounces between full transparency (near the points touching an FT line) to nearly full reflection (near the points touching an envelope NFR). In general, $N_{hs} = O(L/\lambda)$. In an unbound plasma, the solution of (5) with a traveling component, $P > 0$, is a spatially periodic and positively defined, with the intensity, $u^2(\zeta)$, bouncing between two limits, u_{ex}^2, and u_{pk}^2. If $P/16 \ll \delta^2 \ll 1$, we have

$$P = u_{ex}^2 \leq u^2 \leq u_{pk}^2 \approx 8\delta + P/2\delta \tag{9}$$

i. e. the peaks are relatively large, $u_{pk}^2 \gg u_{ex}^2$ and form a train of well separated quasi-solitons nearly coinciding with a standing soliton (8) of the peak intensity $u_{pk}^2 \approx 8\delta$. As P and u_{in}^2 increase, they grow larger and closer to each other. The spatial period, Λ, of this structure is:

$$\Lambda/\lambda \approx ln(16\delta/\sqrt{P})/(2\pi\sqrt{2\delta}) \tag{10}$$

In a strongly RL case, $P > 1$, we have $u_{pk}^2/P \approx 1 + (1+P)^{-2}$, and $\Lambda \approx \lambda/2$, as for a standing, albeit inhibited wave in free space, while traveling wave emerges dominant, resulting in self-induced transparency.

Hysteretic jumps occur when either valley or peak of the intensity profile coincide with an entrance, $\zeta = 0$. The valley, $u_0^2 = u_{ex}^2$, marks an off-jump in Fig. 1, and the peak, $u_0^2 = u_{pk}^2$ – an on-jump. Suppose that in a layer of $L > \lambda/(2\pi\sqrt{2\delta})$, the incident intensity u_{in}^2 is ramped up from zero. When $u_{in}^2 < 2\delta$, Fig. 1. point 1, the amplitude is almost exponentially decaying, $\zeta = 0$, as $u \approx 2u_{in} \exp(-\sqrt{2\delta}\zeta)$, i. e. is a nearly-linear evanescent wave, curve 1 in Fig. 1 inset; the layer is strongly refractive, and the transmission is low. As u_{in}^2 increases, the front end of that profile swells up, becoming a semi-bell-like curve, close to (8) with $\zeta_{pk} \approx 0$. With further slight increase of pumping, it gets unsustainable, and the field configuration has to jump up to the next stable branch of excitation, whereby it forms a steady R-wave at the back of the layer. If after that u_{in}^2 is pulled down adiabatically slow, the R-wave moves to the middle of the layer (Fig. 1. point 2, curve 2 in the inset). Finally, when it is exactly at the midlayer (Fig. 1. point 4, curve 4 in the inset), both valleys are at the borders of the layer, the pumping is nearly minimal to support an R-wave; below it the profile is unsustainable again, and the system jumps down to a regular nearly-evanescent wave and almost full reflection.

At this remarkable point, the layer is fully transparent, i. e. all the (very low) incident power is transmitted through, while a giant R-wave of peak intensity $(u_{in})_{pk}^2 \approx 2\delta$ inside the layer is sustained by a tiny incident power, $(u_{in})_{min}^2$. If $L\sqrt{2\delta}/N\lambda > 1$, the contrast ratio – essentially a nonlinear resonator's finesse, Q, is

$$Q = \frac{(u_{in})_{pk}^2}{(u_{in})_{min}^2} \approx \frac{\exp(2\pi L\sqrt{2\delta}/N\lambda)}{128\ \delta} \gg 1 \tag{11}$$

where N is the number of R-waves in a layer; the one with $N = 1$ occurs after the first jump-up. In the example for Fig. 1 ($\delta = 0.02$, $L = 10\lambda$), $Q \sim 10^7$. In semi-infinite plasma, Q is limited by absorption, see below. It also

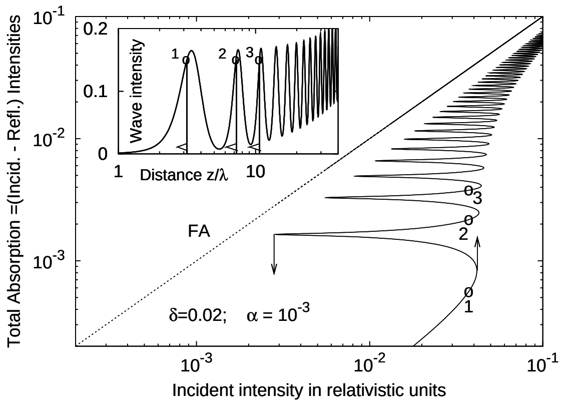

FIG. 2: Hysteretic absorption of light in a semi-infinite plasma layer with absorption α. **FA** is a full absorption limit. Points 1, 2, 3 mark a linear evanescent wave, 1-st, and 2-nd upper stable states respectively. Arrows – the same as in Fig. 1. Inset: intensity profile; points and verticals 1-3 indicate locations of the plasma boundary for the respective points in the main plot; $\triangleleft$'s show direction into plasma layer.

decreases as N increases; the field profile for $N = 2$, is depicted in Fig. 1, point 3, curve 3 in the inset.

Only half of multi-steady-states are stable; the stability condition is that the EM-energy density increases with the pumping, i. e. $dW/d(u_{in}^2) > 0$, which also coincides with the condition $d(u_{ex}^2)/d(u_{in}^2) > 0$, similar to [1].

One can view R-waves at a N-th stable branch as a N-th order mode of a self-induced resonator, with full transparency points marking the resonance. The mirror-like sharp boundaries of plasma layer enhance the resonances [10], but do not constitute necessary condition. Our numerical simulations using (4) showed that a layer with "soft" shoulders making $\sim 50\%$ of the entire layer length, still exhibits a few hysteresises, and a large number of self-induced resonances.

The fundamental manifestation of the phenomenon transpires in a *semi-infinite* plasma. Only two kind of waves [1] in a lossless case satisfy then the Sommerfeld condition – no wave "comes back" from $\zeta \to \infty$ – a traveling (7), $du/d\zeta \to 0$, and an evanescent (8) wave, $u \to 0$. Our investigation of (2), to be published elsewhere, showed that the wave (7) is unstable both in 2D&3D-propagation – and, of (1) – in temporal domain in 1D-case. However, using (4), one can show that even a *steady* 1D-wave (7) is unstable against small absorption by replacing ν^2 in ϵ_{pl} (3) with $\nu^2(1 + i\alpha)$, where $\alpha = (\omega\tau)^{-1}$ is an absorption factor, and τ is an electron momentum relaxation time (typically $\alpha \ll 1$). A condition for a hysteresis to emerge is then $\alpha < \alpha_{cr} \approx e\delta$ [11], if $\delta \ll 1$. Near $\alpha \sim \alpha_{cr}$, a jump-up occurs at $u_{in}^2 \approx \pi\delta$. The ratio (11) to sustain a single R-wave is limited now by $Q \approx \delta/\alpha$, and can still be huge. Hysteresises in reflection at $\alpha = 10^{-3}$ are shown in Fig. 2, and the intensity profile for $u_{in}^2 > 2\delta$ – in the inset. An initially traveling wave develops oscillations due to rising standing wave, which eventually becomes a train of trapped R-waves, the last one being a quasi-soliton close to (8), and then vanishes exponentially. Reducing α pushes that last R-wave further back, but does not extinguish retro-reflection from the R-wave train at the crossover area deep inside plasma, keeping the condition $u \to 0$ at $\zeta \to \infty$.

Lab observation of the phenomena in plasma could be set up with e. g. jets of gas irradiated by a powerful CO_2 laser, with a gas density controlled to reach a crossover point at $\lambda_{CO_2} \approx 10\mu m$; the recent experiments [12] may actually indicate R-waves released by plasma expansion. This process may be naturally occurring in astrophysical environment in plasma sheets expelled from a star (e. g. the Sun); part of the star's radiation spectrum below the initial plasma frequency is powerful enough to penetrate into the layer and be trapped as R-waves. When the layer expands, they can be released as a burst of radiation. It is also conceivable that the R-waves trapping and consequent release may be part of the physics of ball-lighting subjected to a powerful radiation emitted by the main lighting discharge in *mw* and far infrared domains. The R-waves might be used e. g. for laser fusion to deposit laser power much deeper into the fusion pallets; or for heating the ionosphere layers by a powerful *rf* radiation.

In conclusion, optical multi-hysteresises may emerge in a plasma near critical plasma frequency due to fundamental relativistic mass-effect of electrons. They may result in huge trapped, or standing rogue waves with the intensity greatly exceeding that of pumping radiation.

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[1] A. E. Kaplan, JETP Lett. **24**, 114 (1976); also Sov. Physics JETP **45**, 896 (1977).

[2] P. W. Smith, *et al*, Appl. Phys. Lett. **35** 846 (1979); also IEEE JQE, **17** 846 (1981); P. W. Smith and W. J. Tomlinson, IEEE JQE, **20** 30 (1984); S. De Nicola, *et al*, Appl. Phys. B **49**, 441 (1989).

[3] G. A. Mourou, T. Tajima, S. V. Bulanov, Rev. Mod. Phys., **78**, 309 (2006)

[4] W. Chen and D. L. Mills, Phys. Rev. Lett. **58**, 160 (1987), J. E. Sipe and H. G. Winful, Opt. Lett. **13**, 132 (1988), D. N. Christodoulides and R. I. Joseph, Phys. Rev. Lett. **62**, 1746 (1989); B. J. Eggleton, *et al*, *ibid*, **76**, 1627 (1996); D. Mandelik, *et al*, *ibid*, **90**, 053902 (2003).

[5] O. Zobay, S. Potting, P. Meystre, E. M. Wright, Phys. Rev. A **59**, 643 (1999)

[6] A. E. Kaplan, Phys. Rev. Lett. **48**, 138 (1982); *ibid*, **56**, 456 (1986); A. E. Kaplan and P. L. Shkolnikov, Phys. Rev. Lett. **88**, 074801, (2002).

[7] G. Gabrielse, H. Dehmelt, and W. Kells, Phys. Rev. Lett. **54**, 537 (1985).

[8] R. J. Noble, Phys. Rev. A **32**, 460 (1985); B. M. Ashkinadze and V. I. Yudson, Phys. Rev. Lett. **83**, 812 (1999); G. Shvets, *ibid*, **93**, 195004 (2004).

[9] B. Y. Zeldovich, Brief Comm, Lebedev Inst, **2**, 20 (1970).

[10] Weakly nonlinear Fabri-Perot resonator has been used to first demonstrate optical bistability (first-order hysteresis), H. M. Gibbs, S. L. McCall, and T. N. C. Venkatesan, Phys. Rev. Lett. **36**, 1135 (1976); H. M. Gibbs, *Optical Bistability* (Academic Press, New York, 1985).

[11] This suggests an explanation of the hysteresis lacking in [2b]: an artificial nonlinearity used there was due to small dielectric particles suspended in a liquid and had large dissipation because of strong light scattering, *vs* the experiments [2a] and [2c] with nearly transparent fluids.

[12] P. L. Shkolnikov, private communication.